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Abstract

Systems of Systems (SOS) meet vital needs in our society by providing capabilities that are not possible by their discrete components or subsystems. Some SOS are engineered to produce predictable results, yet they can still display emergent behavior. These behaviors are often considered negative because they are not a function of the design. However, emergent behavior can also be serendipitous and produce unexpected positive results. The authors formalize a theory of emergence based on entropy. The theory has explanatory value for emergence as an ontological and phenomenological concept in systems of systems.

Keywords: emergence, entropy, system of systems, semiotics

1. Introduction

Systems of Systems (SOS) meet vital needs in our society by providing capabilities that are not possible by their discrete components or subsystems. These capabilities are manifested in a host of ways that include, but are not limited to: human activities; physical products; informational products; mechanical functions; and logical decisions. These systems are composed of many parts with defined relationships that determine the output of the system.

Let's consider a simplified version of a military system. Managers (high level officers) receive information and make decisions at the macro level to control the actions of agents (boots on the ground) at the micro level. The micro level agents interact with technology, the enemy, civilians, local politicians, the environment, etc. All of these aspects and more, constitute what we have simplified into our "military system" that works to accomplish a common goal (the commander's intent). When information flows freely throughout the system, the order or state of the system (troop configuration, missions, ground actions, equipment location, threat response, etc.) is defined, knowable, and predicable.

A SOS can be engineered to produce such predictable results, but they can also produce unexpected or emergent results. If the cause-effect relationship between all system variables and the output of the system were completely defined, then the SOS would be resultant rather than emergent: properties of the SOS parts would be present in the whole SOS, and most importantly there would be no unexpected behaviors [1] [2]. However, a SOS tends to be a complex system with at least some behaviors that are not completely reducible to system components. Johnson & Tolk [3] posits that in the absence of cause-effect relationships, identifying some otherwise common relationship between SOS micro-level variables (i.e., components) and the macro level states of the SOS, may provide an opportunity to anticipate the onset of emergent behavior.

This paper goes beyond the argument in [3] and develops a formalized theory that there is an information dynamic that reduces the flow of information between the micro and macro levels of the SOS. This leads to an increase in the possible system states (i.e., entropy), and unexpected system outputs (emergence). The implications of this theory could lead to paradigm shifts in risk mitigation strategies, decision analysis, and innovation. The motivation of this paper is to define a theory of emergence based on an information dynamic form of entropy.

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The paper is divided into 6 sections. Section 1 provides the motivations for the paper. In section 2 a brief review of SOS concepts including emergence and entropy. The author's theory of SOS entropy and emergence is constructed in section 3. The theory is demonstrated through example in section 4. The paper concludes in section 5 with observations and opportunities for additional study.

2. System Concepts

In the most basic form, a systems is naively accepted to be a collection of interconnected parts. By this definition almost everything is a system of some sort. A more engineering focused definition includes the concept for an ensemble of autonomous elements, achieving a higher level functionality by leveraging their shared information, feedbacks, and interactions while performing their respective roles [4][5][6][7].

2.1. System of System (SOS)

A SOS is a "super system" or "meta system" comprised of other elements which themselves are independent systems, and interact among themselves to achieve a common goal [8] [9]. Each independent system of a SOS achieves its designated goals whether it is or is not attached to the "super system". A hierarchy of layers exist within the SOS. The hierarchy decomposes into smaller and smaller systems until eventually a level is reached where components in the SOS cannot function independently.

A SOS displays several common themes [10] [11]:

a) The integration of the subsystems results in the performance of higher level mission than that of the members of the meta-system.
b) The integration of the subsystems (or micro level) results in capabilities at the meta-system level (or macro-level) that do not exist at the subsystem level (micro-level).
c) The macro system behavior is emergent.

2.2. Emergence

Emergence, as presented by Keating et al., is not necessarily an absolute concept. There are arguments that emergence exist on a continuum from "Weak" to "Strong" [12] [13] [14]. In the most extreme case, emergence is "Strong" where there is no theory, concept, or principle exist that can explain or deduce the behavior of the system based on the properties or behavior of its micro level components. Emergence is "Weak" in systems to the degree there is some level of traceability between component activity and SOS behavior. Emergence has both ontological and phenomenological meaning:

- Ontological concept (i.e., a characteristic of the system) – As a system characteristic, emergence is the same as irreducibility. The inability to transfer knowledge, methods, causations, or explanations about the macro system to its micro system components, and vice versa [15] [16].
- Phenomenological concept (i.e., observable instance or output of the system) – As an observable instance, emergence is an interesting and unpredicted pattern, behavior or otherwise state of the system [17]. These unexpected results are due to bidirectional interactions between the agents or components the system, and their environment [18] [19] [20] [21]. Whether actually observed or not, the new system state must be observable.

2.3. Entropy

Entropy is an indicator of the tendency of interacting components in a systems to become less correlated and more asymmetrical over time resulting in an increase in the number of system outcomes [22][23]. Stephen et al. theorize that the structure or configuration of the system's parts remains unchanged until entropy reaches a point that it overwhelms the controls of the system and new structures emerge [24]. The degree of order varies on a continuum from completely ordered and predicable to completely unordered and unpredictable (i.e., chaos). As SOS entropy increases and the SOS approaches chaos, the variety of micro and macro states increase, resulting in emergence.

2.3.1 Origins of Entropy

The origins of entropy are rooted in Thermodynamics which concerns the attributes of energy and its transformation from one state to another. In his ninth memoir, the father of entropy (Rudolf Clausius) recounts how he originally "…formed the word entropy…" He states that he intentionally chose a word that sounded like "energy" because the two definitions are so closely related to the condition of a body [25]. Entropy is from a Greek word that means "transformation". It was significant to Clausius to choose a word from an ancient language as an attempt to prevent the meaning from changing over time. Despite Clausius' desire for the meaning of entropy to remain unchanged, the concept today escapes a ubiquitous definition.

2.3.2 Entropy Concepts

Atkins offers an analogous definition of Boltzmann's concept of entropy which is based on the disorder of molecules at various states [26]. Consider the molecules of water. When water is in a solid state (ice) molecules are more tightly ordered than in its liquid state, thus liquid water has higher entropy than ice. When liquid water is boiled and evaporates into a gas state, the molecules are more loosely ordered than when the water was a liquid and even more so than when it was ice. Therefore, the greater the disorder the greater the entropy. This type of disorder might be considered thermal disorder because it is the energy from heat that causes the change in the order of the molecules.

From Atkins' we can also conclude the greater the disorder the greater the number of forms the matter or subject can take on. Molecular arrangement represents order in matter and exist in various degrees:

- Gas = great distance and random arrangement of molecules. Gas assumes the volume and shape of its container.
- Liquid matter = close distance and random arrangement of molecules. Liquids assumes the shape of if container but has an intrinsic volume.
- Solid matter = tightly packed molecules with minimum distance. Solids have an intrinsic shape and volume.
- Heat = energy. As heat is applied to matter, the molecular arrangement changes from maximum order and no emergence (solids) to minimum order and maximum emergence (gas).

Mathematical formulations vary greatly, but there are several distinct common threads that permeate most concepts of entropy. Entropy can be classified into a group of objective physical properties or non-physical (logical) concepts [27]. See Table 1.

Table 1. Entropy Definitions

Entropy Definitions	Physical	Logical
A measure of disorder in a system or an indicator that disorder has occurred [26] [28] [29].	X	
An increasing function of the number of possible states of a system [23]	X	X
The amount of uncertainty associated with the states of a system or a random variable [30].		X
The lack of information about the state of the system [31] [32] [33].		X
A measure of energy quality or efficacy. The amount of energy available to do work [26] [34].	X	
The tendency of interacting components in a systems to become less correlated and more asymmetrical over time [22].		X

3. Theory of SOS Entropy and Emergence

We theorize that the combination of changing information and system disorder, is a special case of entropy applicable to system of systems (i.e., SOS entropy). Here the increasing information (energy) leads to increased disorder (entropy) which causes macro system behaviors and new micro system relationships (emergence). Restating the theory: as SOS entropy increases, the variety of micro relationships and macro states increase, resulting in emergence. The transition between forms and spatiotemporal patterns described by this theory is governed by the rate of entropy production [35]. There are four aspects of entropy in a SOS that help explain this dynamic: information; semiotics; requisite variety; dispersal and decay.

3.1. Information.

Information is the commodity that allows agents to function with other agents and with the macro system. It is the energy that enables interaction between agents. Information are actionable interpretations of data or other facts about a noun (a person, place, thing, or idea) that is gained through study, experience, or by instruction [36] [37] [38]. Information has meaning and is recognizable by its receiver. It can take on many forms and includes social norms; attitudes; outputs form media sources (fax, email, phone, TV, radio, internet, intranet, etc.); feedback and response to errors and stress (consequences, pain, etc.). Information allows the receiver to reduce uncertainty; draw distinctions; identify similarities; justify beliefs; and make decisions. There is also a dynamic component in this definition. To gain something suggest that is was previously absent at time t0 and present at time tx. However, information is not strictly a matter of addition to what already exist. Deacon, describes the nature of information as defining the relationship between something that is present and something that is absent [22]. This is true whether the expectation is due to regularity of previous occurrences or habits of the agent. The absence or omission of something that is expected, leads to actions (work); is an indication that some work has occurred; or that some work has not occurred [39]. These indications of changes in the absence or presence of energy are indications that entropy is increasing or decreasing in the system.

3.2. Semiotics.

In order for subsystems (or agents) in a SOS to communicate, they must have the ability to share information. The information exchange agreements between the agents in a SOS can be characterized by a three –tuple of: 1) syntactic symbols; 2) semantic definitions of the symbols; and 3) pragmatic use of symbols [40]. These exchanges agreements allow agents in the system to identify or sense information based on what is present as wells as what is absent. Tolk et al. suggest that the possible solutions (i.e., emergent solutions) produced by models can be optimized by managing the semiotic aspects to achieve desired levels of entropy.

3.3. Requisite Variety.

The range of outputs from subsystem activities include those that are desired and those that are not. The SOS includes a set of subsystems (i.e., regulators) whose roles are to control the outputs of other subsystems by limiting the undesired outputs and allowing the desired outputs. Regulators must be able to receive information about the state of the subsystems the control and be able to exchange information with other regulators. Based on the capacity of each regulator to exercise control, there is a minimum ratio (i.e., variety) of regulating to regulated subsystems that is required for the SOS to achieve its objective or perform its intended function [41]. As the actual variety becomes less than the required or requisite variety of the SOS, regulators are not able to obtain and exchange the requisite information to control the behavior in the SOS, and the system becomes emergent.

3.4. Dispersal and Decay.

The semiotic attributes of a SOS enable the capability for subsystems to recognize each other and exchange information (i.e., communicate). Recognition and information exchange is a requirement for the subsystems to be able to generate interactions and coordinated efforts [31]. Potential subsystem interactions increase (i.e., disperse) and decrease (i.e., decay) as a direct function of the extent to which the recognition and information exchange capability exists the SOS. As subsystems are able to recognize and exchange information with each other, the SOS becomes more emergent.

3.5. Conceptual Framework

Information is the common thread that drives increasing disorder due to dispersal and decay or to insufficient requisite variety. Information in this context serves the same purpose and function as energy in Rudolf Clausius's quantitative definition of entropy as generalized by Peter Atkins [26]. The authors adapt this definition and substitute information for energy in Clausius's definition. From the requisite variety concept, we are only concerned with information that is required for controlling behavior of the SOS (i.e., requisite information). Whether defined as degree of disorder or a function of state probabilities, entropy is an additive function of independent systems within a SOS [28] [42]. From these points, SOS entropy is defined as follows:
Where time T is $\{T_0, T1, T2, T3...Tx\}$,

$$\textit{Clausius's Entropy = Energy (heat) at Time } T_X \div \textit{Initial Energy (temperature) at Time } T_0 \qquad (1)$$

Substituting requisite information for energy as heat,

$$\textit{Entropy (S) = Requisite Information at Time } T_X \div \textit{Requisite Information at Time } T_0 \qquad (2)$$

Where the SOS consists of subsystems $\{1, 2, 3...n\}$,

$$\textit{SOS Entropy = S1+S2+S3+ ...Sn.} \qquad (3)$$

This is a conceptual framework only. SOS Entropy as an additive function may be more appropriate for a federation of systems (FOS) where each system is truly independent [43]. For other constructs of SOS, entropy may very well be the result of interactions between subsystems. In this case, SOS Entropy is approximated by equation 4:

$$\textit{SOS Entropy = C x (S1xS2xS3x...Sn)} \qquad (4)$$

Where C is some constant <0.

A specific method to determine requisite information at T0 and TX still needs to be determined. However, we can explore the phenomenological aspects theory in the context of an example.

4. For An Example "Look to the Ant"

In many ways, SOS behaviors are similar to behaviors found in natural systems. For an example that demonstrates the relationship between entropy and emergence, we need look no further than the ant [44] [45]. An ant colony is a multi-agent system (a type of SOS). The dynamic relationship between the ants, the colony, and their environment can be studied for insights to SOS [45].

4.1. Ant Colonies

Ant colonies are complicated systems where the efforts of thousands of autonomous agents successfully accomplish a single mission; the growth and survival of the colony. Each ant and the ant colony is a four-tuple agent: state, inputs, outputs, and ontology. The environment in which the agent operates is a two-tuple: state, and ontology.

The individual ants, the colony, and the environment are coupled through their semiotics (i.e., ontologies). These ontologies define the rules and specifications for how the three entities interact and dynamically change states [46] [47]. A simplified description of the ant colony system is basically:

a) Ants are either foraging for food or bringing food to the nest (i.e., nesting). They deposit pheromone markers along the path that indicate their origin (nest or food). This allows ants going in the opposite direction to find their way to their destination.
b) Ants want to find the shortest path to food and to their nest while avoiding hazards. The ants walk a random path seeking the pheromones deposits of other ants (a scent trail) to find their way to food and back to their nest.
c) The pheromones evaporate into the environment allowing the ants to sense the general direction but not the specific location of the deposit. The strength of the scent also declines over time.
d) The environment provides food as well as obstacles and hazards that work against foraging and nesting efforts.
e) The macro-level order in the colony system is represented by the network of paths (number of paths and the coordinate maps for each path). The micro-level order in the system is represented by the ant's activity and location coordinate.
f) Once food is located and the first foraging/ nesting cycle is completed, the initial path is established. As more ants complete the foraging/nesting cycle, the number of specific paths in the network increase.

4.2. Ant Colony Entropy

The information dynamics in the ant colony example are the deposits and evaporation of pheromones; the addition of hazards; and changes in food supply. The change in interaction between the ants due to the increasing and decreasing strength of the pheromones represents the dispersal and decay process for entropy. The requisite variety concept is not apparent in this example. However, we can consider the foraging ants as regulator for the nesting ants (and vice versa). Each role controls the path of the next role. A balance between forging and nesting ants is necessary or the required information (i.e., pheromones) would not be available and the paths between nest and food would become less efficient (if not random).

Initially the population of ants that has completed the forging/ nesting path is low, there are minimum pheromone deposits, and no new obstacles. Entropy is low, number of paths are few, and path predictability is high. Over time, more ants complete the path cycle, interactions and bidirectional information exchanges between the ants increase. The results are the ant population grows; even more ants complete the foraging /nesting cycle; more pheromones are emitted into the environment; the environment creates hazards; food sources are depleted and new food sources are found; ants adjust their path in response to requisite information (pheromones, hazards, and changes in food supply).

The increasing information dynamic caused by the interactions of the ants and their environment leads to growing disorder (i.e., entropy) in the ant colony system: less predictability of exact coordinates of ant paths; and changing configuration of the colony's network of paths. See **Error! Reference source not found.**. The blue lines represent the paths from the nest to the food and the red lines represent the paths form the food to the nest.
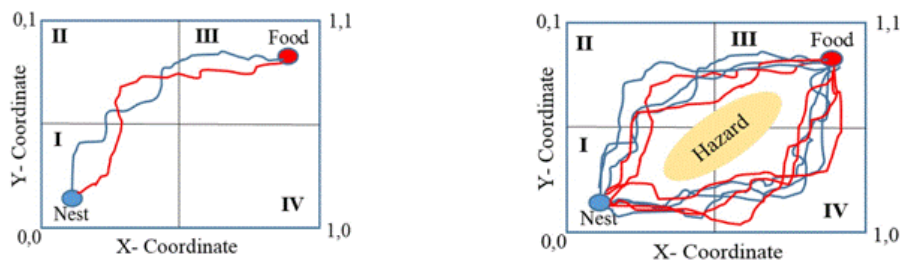

Figure 1. (a) Initial Forging/Nesting Path Network at Time = t1; (b) Forging/Nesting Path Network at Time = tx

4.3. Ant Colony Emergence.

Although the ontology in the ant colony system guides the path decisions, the actual path is stochastically determined.   The reference modes for the ant colony SOS is presented in equation (4) and Figure 1.

$$\uparrow Time \Rightarrow \uparrow Information \Rightarrow \uparrow Interactions \Rightarrow \uparrow Entropy \Rightarrow \uparrow Disorder \Rightarrow \downarrow Path\ Predictability \Rightarrow \uparrow Emergence \quad (4)$$

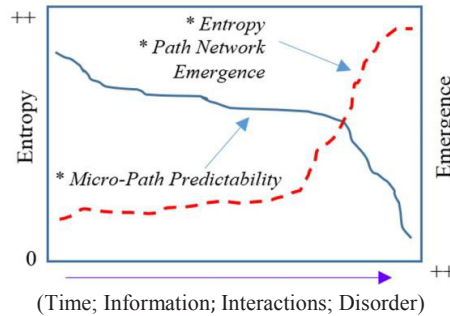(Time; Information; Interactions; Disorder)

Figure 1. Ant Colony Reference Modes

5. Conclusions

An analysis of the qualitative data from the literature review points to a theory for the relationship between entropy and emergence in SOS. Entropy $\Rightarrow$ SOS Emergence is such that as SOS entropy increases and the SOS approaches disorder, the variety of micro and macro states increase, resulting in emergence.   The SOS entropy emergence theory is supported through correspondence with a natural SOS (i.e., the ant colony). The literature review and the SOS example support the belief that rising entropy precedes emergence and is therefore a leading indicator of emergent states and events.   The theory further explains the behavior of a SOS in terms of entropy as an information dynamic.

There are several opportunities to improve the theory and its presentation.   The formulation for information, entropy, and emergence in section 3.5 are useful as a conceptual frameworks.   However, additional work is needed to determine how to aggregate the entropy of subsystems into the entropy of the SOS.   The authors demonstrate the theory through an example of emergence as a phenomenon. Examples are also needed to demonstrate the theory in an ontological emergence.

Additional research is underway as the authors attempt to determine if entropy has a causal relationship with emergence. The authors are creating an experimental setting by constructing a conceptual SOS model and simulating it to test the coherence of the theory with other theories and accepted beliefs about entropy and emergence in SOS.

References

1. Bar-Yam, Y. (2004). A mathematical theory of strong emergence using multiscale variety. Complexity, 9(6), 15-24.
2. Goldstein, J. (1999). Emergence as a construct: History and issues. Emergence, 1(1), 49-72.
3. Johnson, J., & Tolk, A. (2013). Entropy: An Indicator Of Emergence In System Of Systems. Proceedings of the American Society for Engineering Management 2013 International Annual Conference.
4. INCOSE (2011). Systems Engineering Handbook: A Guide for System Life Cycle Processes and Activities; International Council on Systems Engineering (INCOSE). INCOSE-TP-2003-002-03.2. 1.
5. Zadeh, L. A. (1969). The concepts of system, aggregate, and state in system theory.
6. Buede, D. M. (2011). The engineering design of systems: Models and methods (Vol. 55). Wiley.
7. Sokolowski, J., & Banks, C. M. (2010). Modeling and simulation fundamentals: theoretical underpinnings and practical domains. Wiley.
8. Clark, J. O. (2009, March). System of systems engineering and family of systems engineering from a standards, V-model, and dual-V model perspective. In Systems Conference, 2009 3rd Annual IEEE (pp. 381-387). IEEE.
9. Karcanias, N. I. C. O. S., & Hessami, A. G. (2010, September). Complexity and the notion of system of systems: Part (II): defining the notion of system of systems. In World Automation Congress (WAC), 2010 (pp. 1-7). IEEE.
10. Keating, C. B. (2008). Emergence in system of systems. System of Systems Engineering, 169-190.
11. Keating, C., Rogers, R., Unal, R., Dryer, D., Sousa-Poza, A., Safford, R., & Rabadi, G. (2003). System of systems engineering. Engineering Management Journal-Rolla-, 15(3), 36-45.
12. Chalmers, D. J. (2006). Strong and weak emergence. The reemergence of emergence, 244-256.
13. Bar-Yam, Y. (2004). A mathematical theory of strong emergence using multiscale variety. Complexity, 9(6), 15-24.
14. Bedau, M. A. (1997). Weak emergence. Noûs, 31(s11), 375-399.

15. Christen, M., & Franklin, L. R. (2002). The concept of emergence in complexity science: Finding coherence between theory and practice. Proceedings of the Complex Systems Summer School, 4.
16. Menzies, P. (1988). Against causal reductionism. Mind, 97(388), 551-574.
17. Holland, O. T. (2012). Partitioning method for emergent behavior systems modeled by agent-based simulations. Old Dominion University). ProQuest Dissertations and Theses, 283. Retrieved from http://search.proquest.com/docview/1283121663?accountid=12967.
18. Odell-a, J. (2002). Agents and complex systems. Journal of Object Technology, 1(2), 35-45.
19. Odell-b, J. (2002). Objects and agents compared. Journal of object technology, 1(1), 41-53.
20. Parunak, V., Bruekner, S., & Sauter, J. (2002, January). ERIM's approach to fine-grained agents. In NASA/JPL Workshop on Radical Agent Concepts, Greenbelt (MD).
21. De Wolf, T., & Holvoet, T. (2005). Emergence versus self-organisation: Different concepts but promising when combined. Engineering self-organising systems, 77-91.
22. Deacon, T. W. (2007). Shannon–Boltzmann–Darwin: Redefining information (Part I). Cognitive Semiotics, 2007(15), 123-148.
23. Adriaans, P. (2012). "Information", The Stanford Encyclopedia of Philosophy (Winter 2012 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/archives/win2012/entries/information/
24. Stephen, D. G., Dixon, J. A., & Isenhower, R. W. (2009). Dynamics of representational change: Entropy, action, and cognition. Journal of Experimental Psychology: Human Perception and Performance, 35(6), 1811.
25. Clausius, R. (1867). The mechanical theory of heat: with its applications to the steam-engine and to the physical properties of bodies. J. van Voorst.
26. Atkins, P. (2003). Galileo's Finger: The Ten Great Ideas of Science: The Ten Great Ideas of Science. OUP Oxford.
27. Carnap, R., & Shimony, A. (1977). Two Essays on Entropy. Univ of California Press.
28. Chakrabarti, C. G., & De, K. (2000). Boltzmann-Gibbs entropy: axiomatic characterization and application. International Journal of Mathematics and Mathematical Sciences, 23(4), 243-251.
29. Bradnick, D. (2008). A Pentecostal Perspective on Entropy, Emergent Systems, and Eschatology. Zygon: Journal Of Religion & Science, 43(4), 925. doi:10.1111/j.1467-9744.2008.00969.
30. Cover, T. M., & Thomas, J. A. (2006). Elements of information theory. Wiley-interscience.
31. Testa, B. (2009). Dispersal (Entropy) and Recognition (Information) as Foundations of Emergence and Dissolvence. Entropy, 11(4), 993-1000. doi:10.3390/e11040993
32. Machta, J. (1999). Entropy, information, and computation. American Journal of Physics, 67, 1074.
33. Jaynes, E. T. (1957). Information theory and statistical mechanics. Physical review, 106(4), 620.
34. Atkins, P. (2007). Four laws that drive the universe. OUP Oxford.
35. Frank, T. D. (2011). Rate of Entropy Production as a Physical Selection Principle for Mode-Mode Transitions in Non-Equilibrium Systems: With an Application to a Non-Algorithmic Dynamic Message Buffer. European Journal of Scientific Research, 54(1), 59-74.
36. Information (2013). Oxford Dictionaries Online, Oxford University Press. Retrieved from URL = http://oxforddictionaries.com/us/definition/american_english/information?q=information
37. Losee, R. M. (1997). A discipline independent definition of information. JASIS, 48(3), 254-269.
38. Buckland, M. K. (1991). Information as thing. JASIS, 42(5), 351-360.
39. Deacon, T. W. (2008). Shannon-Boltzmann-Darwin: redefining information (Part II). Cognitive semiotics, 2008(2), 169-196.
40. Tolk, A., Diallo, S. Y., & Padilla, J. J. (2012). Semiotics, entropy, and interoperability of simulation systems: mathematical foundations of M&S standardization. In Proceedings of the Winter Simulation Conference (p. 243). Winter Simulation Conference.
41. Ashby, W. R. (1958). Requisite variety and its implications for the control of complex systems. Cybernetica, 1(2), 83-99.
42. Shannon, C. E. (2001). A mathematical theory of communication. ACM SIGMOBILE Mobile Computing and Communications Review, 5(1), 3-55.
43. Sage, A. P., & Cuppan, C. D. (2001). On the systems engineering and management of systems of systems and federations of systems. Information, Knowledge, Systems Management, 2(4), 325-345.
44. Parunak, V., & Brueckner, S. (2001, May). Entropy and self-organization in multi-agent systems. In Proceedings of the fifth international conference on Autonomous agents (pp. 124-130). ACM.
45. Dorigo, M., & Gambardella, L. M. (1997). Ant colony system: A cooperative learning approach to the traveling salesman problem. Evolutionary Computation, IEEE Transactions on, 1(1), 53-66.
46. Parunak, H. (1997). " Go to the ant": Engineering principles from natural multi-agent systems. Annals of Operations Research, 75, 69-102.
47. Gruber, T. (2008). What is an Ontology? Encyclopedia of Database Systems, 1.